

Experimental quantum cosmology in time-dependent optical media.

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Abstract. It is possible to construct artificial spacetime geometries for light by using intense laser pulses that modify the spatiotemporal properties of an optical medium. Here we theoretically investigate experimental possibilities for studying spacetime metrics of the form $ds^2 = c^2 dt^2 - \eta(t)^2 dx^2$. By tailoring the laser pulse shape and medium properties, it is possible to create a refractive index variation $n = n(t)$ that can be identified with $\eta(t)$. Starting from a perturbative solution to a generalised Hopfield model for the medium described by an $n = n(t)$ we provide estimates for the number of photons generated by the time-dependent spacetime. The simplest example is that of a uniformly varying $\eta(t)$ that therefore describes the Robertson-Walker metric, i.e. a cosmological expansion. The number of photon pairs generated in experimentally feasible conditions appears to be extremely small. However, large photon production can be obtained by periodically modulating the medium and thus resorting to a resonant enhancement similar to that observed in the dynamical Casimir effect. Curiously, the spacetime metric in this case closely resembles that of a gravitational wave. Motivated by this analogy we show that a periodic gravitational wave can indeed act as an amplifier for photons. The emission for an actual gravitational wave will be very weak but should be readily observable in the laboratory analogue.

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1. Introduction.

Analogue gravity is the study of gravitational systems, or more in general of curved spacetimes by means of analogue systems whose kinematics, although based on different underlying physical equations (i.e. dynamics), are governed by identical or similar spacetime metrics [1, 2, 3]. So, whilst it is not possible to use these systems to mimic for example the precise dynamics and interaction between gravitating bodies, it is possible to reproduce the physics of phenomena that rely solely on the specific shape of the spacetime metric. The most studied phenomenon in this sense is without doubt Hawking radiation, i.e. the emission of photons excited out of the vacuum state in the proximity of an event horizon. Originally predicted by S. Hawking to occur for static black holes [4], this prediction was later extended by W. Unruh to analogue systems [5], i.e. to acoustic waves propagating in a fluid with a flow that reproduces the same flow of space falling into a black hole. Since this seminal work, many different proposals have been theoretically developed relying on system as varied as Bose-Einstein Condensates, gravity waves in water, microwave radiation in waveguides and ultrashort laser pulse propagation in optical media thus highlighting the ubiquity and general nature of Unruh's idea (see e.g. [1, 2, 3] for an overview). However, it is only in recent years that some of these ideas reached experimental maturity with the first serious attempts to actually create artificial horizons and observe Hawking radiation [6, 7, 8], including the actual realisation of Unruh's original idea of a horizon in a flowing fluid [9].

Clearly, the same rationale of reproducing a spacetime metric and therefore the kinematics with any related quantum behaviour applies not only to black holes but also to other scenarios. Examples are the Unruh effect, a cosmological expansion [10], metric signature changes [11] or, going beyond cosmology, the dynamical Casimir effect [12].

Whilst there are many possible different systems in which to build these analogues, here we focus attention on the specific case of the spacetime metric induced by an ultrashort and intense laser pulse propagating through a thin film of material. Leonhardt and co-workers first proposed a similar system, a laser pulse propagating through an optical fibre, as a way to recreate a horizon [13]. At sufficiently high intensities the laser pulse will excite a nonlinear response in the medium (described by the nonlinear index n_2) that in turn creates a local variation of the refractive index, n_0 , that can be written as $n(z - vt) = n_0 + n_2 I(z - vt)$, where z is the propagation direction and v the propagation velocity of a pulse with intensity $I(z - vt)$. The horizon is generated in the comoving frame of the laser pulse where the local increase in refractive index $\delta n = n_2 I$ can slow down a co-propagating light pulse and effectively block it [8]. Theoretical analysis of this system shows that it is described by the Gordon metric $ds^2 = (c/n)^2 dt^2 - dz^2$ [14], which may be re-written in a similar form to the Painlevé-Gullstrand metric for a black hole, $ds^2 = c^2 dt^2 - (dz - v dt)^2$ [15]. Some simplifying assumptions are made in this derivation, e.g. we neglect optical dispersion and also work in the geometrical optics limit so that

the magnetic permeability μ is constant and the optical analogue is created by acting only upon the dielectric permeability $\varepsilon = n^2$. Nevertheless, numerical simulations based for example of the direct solution of the full Maxwell equations verify the behaviour of the laser induced δn in terms of creating a blocking horizon and also the scattering of input waves into two output modes that can be identified with the outgoing positive and infalling negative-energy Hawking modes generated at the horizon [16].

Here we expand on this system based on optical nonlinearity to examine a slightly different class of spacetime metrics, namely metrics in which $n = n(t)$, i.e. a medium in which the refractive index varies as a whole. Maxwell's equations are conformally invariant so that we may transform the corresponding spacetime metric into the form

$$ds^2 = c^2 dt^2 - n(t)^2 dx^2 \quad (1)$$

where x is a transverse spatial coordinate. This metric encompasses a large class of different cosmological settings depending on the specific form we choose for $n(t)$, ranging from the Robertson-Walker metric [17] for a cosmological expansion to gravitational waves.

The physical system we are studying is schematically depicted in Fig. 1: a thin, square film of a transparent nonlinear material, e.g. graphene is subject to a laser pulse or series of laser pulses directed along the z -axis that uniformly illuminate the thin film. The nonlinear response excited by the laser beam creates the time-dependent refractive index $n(t)$.

In the following we will first describe the model that we use to study photon production from the vacuum state in the presence of the time-varying refractive index. We then apply this model to two specific cases represented by the metric (1), namely a cosmological expansion (or contraction) and a periodic spacetime expansion-contraction. The latter metric may be likened to a gravitational wave and, motivated by this analogy, we show that indeed a gravitational wave may be expected to act as an amplifier for photons.

2. Numerical model

The main objective of this work is to provide a prediction for the number of photons that will be excited from the vacuum state due to a generic $n = n(t)$ time variation of the refractive index of a given medium. In order to do this, we rely on a very general framework based on the Hopfield model that allows us to describe the medium as a collection of oscillators and from this, after performing a perturbative expansion under the assumption that the δn amplitude is small with respect to the background, constant index n_0 and therefore evaluate the number of photons in the system after the refractive index change with respect to the initial, stationary input state. The full details of this model are given in Ref. [18] to which we refer the reader and here we just summarise the main equations.

The medium response is described via a Hopfield model as a set of harmonic oscillators

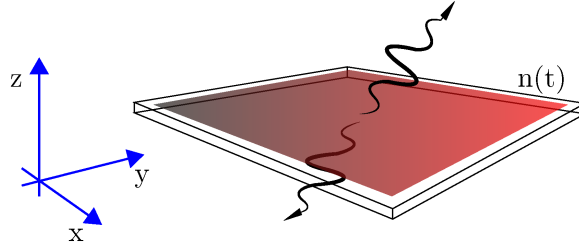


Figure 1. Schematic layout of the system analysed in this work. A thin film of a transparent nonlinear material, e.g. graphene, with thickness L along the z -axis is subject to a laser pulse or series of laser pulses directed along the z -axis and uniformly illuminate the thin film. The nonlinear response excited by the laser beam creates a time-dependent refractive index $n(t)$. The interaction of the time varying $n(t)$ with the surrounding background vacuum states will lead to the emission of photon pairs.

that are coupled to the quantised electromagnetic field by a coupling constant. The full response of the actual medium is described by the coupled dynamics of the bare electromagnetic field with the medium fields. The stationary (non time-varying medium) is described by a Hamiltonian that contains the medium polarization as sum over N resonant oscillator contributions that correspond to the physical resonances that characterise all dispersive media and fully determine the dispersive properties. The medium is therefore characterised by a generic dispersion relation of the form

$$c^2 k^2 = \omega^2 \left[1 + \sum_{l=0}^N \frac{\chi_l}{\omega_l^2 - \omega^2} \right]. \quad (2)$$

Most dielectric media typically have one or two poles located in the ultraviolet or deep-ultraviolet part of the spectrum (e.g. located at wavelengths shorter than 100-200 nm) and an additional resonance in the mid or far-infrared region (e.g. located at wavelengths longer than 5-10 μm). In the following we will refer to a simplified model material in which we consider only a single resonance [$N = 0$ in Eq. (2)]. This is justified in light of the fact that we will be examining the behaviour of a limited spectral range in the visible and near infrared region that is indeed dominated in most media of interest by the UV resonance. Moreover, although it is possible to extend the Hopfield model to account for both dispersive and absorptive properties of the medium, here we only consider the dispersive component and neglect any effects due to absorption, a justifiable approximation as long as the electromagnetic field frequencies involved in the interactions are far from the resonance frequencies ω_l . This is readily verified in common experimental conditions - for example, fused silica glass exhibits a resonance in the UV region located at ~ 200 nm but is essentially transparent to wavelengths longer than 300 nm.

Spacetime distortions in the medium are then modelled as a perturbation to the χ_0 parameter, i.e. we use $\chi(x, t) = \chi_0 + \delta\chi(x, t)$. The spacetime perturbation is related to the laser pulse induced δn by the relation $\delta\chi(x, t) = 2(\omega_0^2 - \omega^2)n_0\delta n(x, t)$. Therefore, by controlling the laser pulse and the interaction geometry with the medium, we have

direct control over the $\delta\chi(x, t)$ and hence an experimental handle with which to control the photon emission from the modulated medium.

In the perturbative limit the number of photon pairs excited by the spacetime varying medium is determined by the scattering matrix $S \simeq \mathbb{1} - (i/\hbar) \int dx dt \delta H(x, t)$, where $\delta H(x, t)$ is the perturbation to the background Hamiltonian. From this it is possible to calculate the number of photon pairs emitted per unit solid angle, with frequencies ω_1 and ω_2 , from the relation

$$\frac{dN}{dt} = f(\omega_1, \omega_2) \times \left| \widehat{\delta\chi}(\omega_1 + \omega_2, k_1 + k_2) \right|^2 d\omega_1 d\omega_2 \quad (3)$$

where $\widehat{\delta\chi}$ denotes the Fourier transform of $\delta\chi$ and $f(\omega_1, \omega_2)$ is a function of the photon pair frequencies

$$f(\omega_1, \omega_2) = \frac{1}{4(2\pi c)^6} \frac{\omega_1^2 \omega_2^2 (\omega_1 \omega_2 + \omega_0)^2}{(\omega_0^2 - \omega_1^2)^2 (\omega_0^2 - \omega_2^2)^2} \omega_1 n(\omega_1) \omega_2 n(\omega_2). \quad (4)$$

The main result of this model is therefore that the number of photon pairs depends essentially on the Fourier transform of the refractive index perturbation, parametrized in the model by the resonance oscillator strength $\delta\chi$: it is $\widehat{\delta\chi}$ that determines the precise emission pattern and number of the photon pairs [18].

In the following we proceed to show some specific examples in which this model predicts the generation of photon pairs in time varying spacetimes of the form given in Eq. (1).

3. Photon production from an artificial expanding (contracting) universe.

Previous studies have outlined how the metric Eq. (1) may be identified with the one dimensional version of the Robertson-Walker metric that governs the kinematics in an expanding or contracting Universe. These studies examined the behaviour of media that reproduce this metric from a classical perspective, e.g. an analysis was given of the red-shift imparted upon a classical probe laser beam. Here we extend this analysis to incorporate quantum effects and estimate the rate of photon pair production from an artificial expanding universe.

From an experimental perspective, the challenge is to create a medium in which the expansion is indeed constant, i.e. such that $n(t)$ is a continuously increasing or decreasing function of time. Any attempt to obtain this over long time scales will lead to either huge or vanishing refractive indices, respectively and are therefore likely to fail. Moreover, as we will see below, the rate at which the artificial universe expands or contracts, i.e. the time gradient of $n(t)$ is also important. However, the expansion or contraction of the medium does not need to proceed indefinitely and the only requirement is that it is uniform and rapid over the time scale of a given test pulse that is used to probe the metric or, in our case, over the time scale over which the photon pairs are generated. In short, we propose an experimental setup similar to that shown in Fig. 1 in which a very thin and highly nonlinear medium is pumped by an intense, ultrashort laser pulse. For example, we may use gold or graphene that have very high optical nonlinearity, n_2 , deposited either in a thin film or in a multilayer

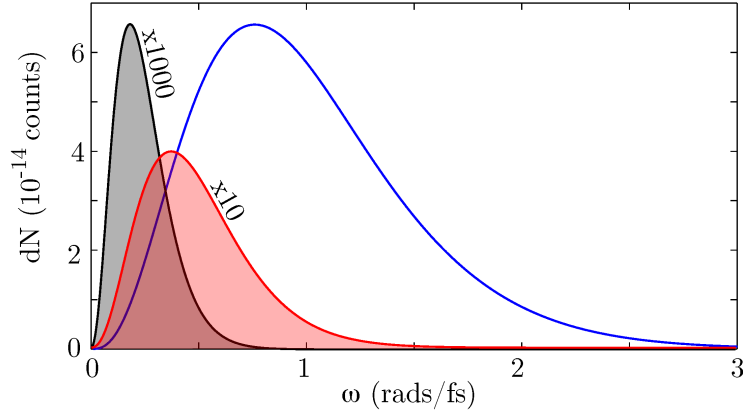


Figure 2. Numerical estimates of the emitted photon numbers for $n(t) = \tanh(\alpha t)$, with $\alpha = 20 \times 10^{13}$ 1/s (blue curve), $\alpha = 40 \times 10^{13}$ (red shaded curve, multiplied by 10), $\alpha = 80 \times 10^{13}$ (black shaded curve, multiplied by 1000).

structure with an overall thickness that can be of the order of a few nms up to a micron. The pump pulse that excites the nonlinearity should have a rise time that is as short as possible yet larger than the film thickness in order to ensure that there are time transients during which only the leading edge (giving rise to an continuously increasing $n(t)$) or trailing edge (giving rise to an continuously decreasing $n(t)$) overlap with the film.

As an example we choose $\delta\chi(t) = \chi_0 \tanh(\alpha t)$ with the amplitude χ_0 chosen such that the maximum refractive index variation in the medium is 10^{-2} and we study the emission for various rise times $\tau = 1/\alpha = 1.25, 2.5$ and 5 fs. Results are shown in Fig. 2 for the specific case in which the photons are emitted back to back along the z direction, i.e. perpendicular to the plane of the thin film. We found that this configuration gave an emission that was 2 or more orders more efficient than other configurations e.g. two photons emitted in the forward direction or photons emitted in the x - y plane (data not shown).

The first observation is that the model does indeed predict the creation of photon pairs from the time-varying medium thus supporting the idea that our artificial expanding universe interacts with the vacuum by creating new photon states. The emitted photons also show a clear frequency dependence with a peak emission that increases with increasing α , i.e. faster changes of rise times τ of the $n(t)$ lead to higher frequency photons and also higher photon count rates.

As can be seen, even if we have chosen a very large δn , the actual number of photons is extremely small and very unlikely to be detectable. The total photon numbers emitted per second obtained by integrating over the whole frequency range are $N \simeq 10^{-2}, 10^0$ and 10^2 for $\tau = 5, 2.5$ and 1.25 fs, respectively. These numbers are then further reduced by ~ 15 orders of magnitude due to the fact that emission is stimulated only for the duration of the $n(t)$ rise time, τ . Reducing the δn to more reasonable levels, e.g. $10^{-3} - 10^{-4}$ worsens this situation even further. We also verified that changing the specific shape of

the $\delta\chi(t)$, e.g. to a Gaussian pulse shape, does not appreciably change this result. However, in the next section we propose a method by which the photon emission mechanism examined here may be resonantly enhanced to experimentally detectable levels.

4. Resonant enhancement of photon production by periodic contraction-expansion.

In order to enhance the photon emission whilst maintaining the same dielectric medium geometry shown in Fig. 1, we consider the case in which the laser pump pulse is actually a periodic train of pulses such that we may model the medium response with a

$$\delta\chi(t, x, y) = \kappa H(x, y) \cos(\Omega t) \quad (5)$$

where $H(x, y)$ is a function that defines the shape of the film and Ω is the oscillation period of the laser pulse train and hence also of the medium. Experimentally it is possible to create a train of laser pulses by super-imposing long or even continuous wave laser beams with different frequencies. In the case in which we use frequencies that are harmonics of some fundamental wavelength, it is actually possible to perform Fourier synthesis in the optical domain and create laser pulse trains with arbitrary shapes. The great advantage of this technique, with respect e.g. to resorting to a standard pulsed laser system is that the pulse periodicity may be driven to extremely high values. For example, combining two beams with frequencies ω_a and ω_b delivers a waveform with an envelope that oscillates with a periodicity of $\Omega = (\omega_a - \omega_b)$ - if we choose to use a standard Nd:Yag laser (fundamental wavelength, 1064 nm) and its fourth harmonic as the two generating beams, then $\Omega = 5.3$ rads/fs which is more than six orders of magnitude larger than what is obtained from standard “high” repetition rate lasers that will typically operate at a maximum rate in the range of 100 MHz.

The Fourier transform of $\delta\chi$ determines the properties of the emitted photons. For the specific case in which the film is square with side length L , as previously considered we have

$$\begin{aligned} |\widehat{\delta\chi}(\omega_1 + \omega_2, \mathbf{k}_1 + \mathbf{k}_2)|^2 &= \\ &= \frac{\pi}{2} \kappa^2 \delta(\omega_1 + \omega_2 - \Omega) L^4 \left| \text{sinc} \left(L \frac{k_{1x} + k_{2x}}{2} \right) \right|^2 \left| \text{sinc} \left(L \frac{k_{1y} + k_{2y}}{2} \right) \right|^2 \end{aligned} \quad (6)$$

Interestingly, the introduction of a periodic modulation introduces a δ function that now strongly conditions the emitted frequencies, i.e. resonance with the laser pulse train periodicity limits the photon emission to frequencies given by $\omega_1 + \omega_2 = \Omega$. This condition is strongly reminiscent of photon emission by the dynamical Casimir effect, i.e. by periodic modulation of a cavity. In our case there is no cavity, yet the periodicity of the modulation imposes similar resonance conditions.

Integration of Eq.s (3) and (6) provide us with the photon number estimates shown in Fig. 3 from a film with thickness 200 nm for the case of back to back emission in the x (or y) direction [Fig. 3(a)] and z direction [Fig. 3(b)]. The different film sizes

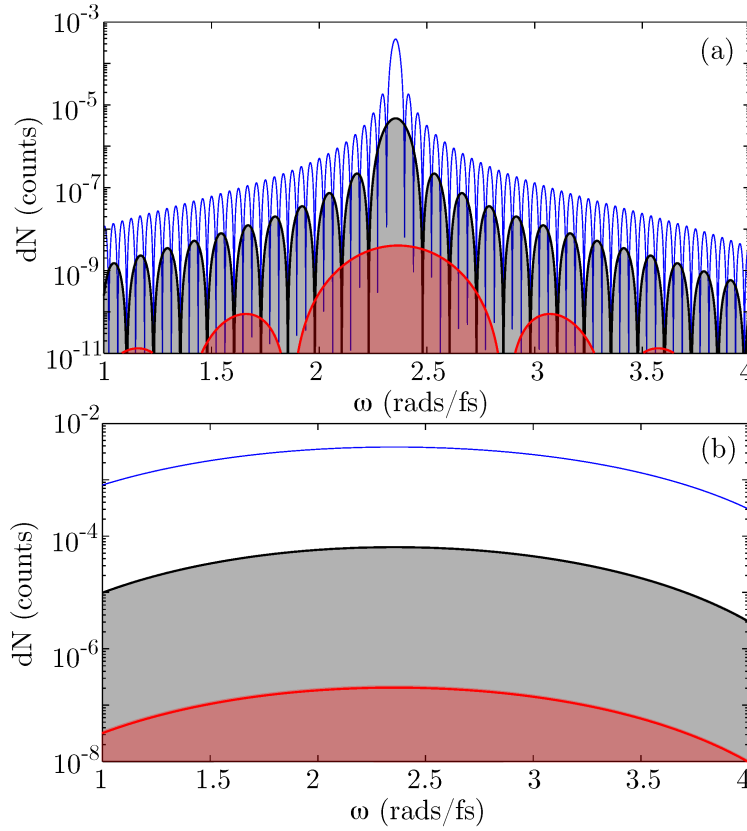


Figure 3. (a) Number of photon pairs for back to back emission in the x direction (transverse to the thin film). (b) Number of photon pairs for back to back emission in the z direction (perpendicular to the thin film). In each graph, three different square film sizes are considered, with side lengths of 2 (blue line), 6 (black line, grey shaded area) and 20 μm (red line, red shaded area).

are considered, i.e. square films with side length $L = 2$ (red, shaded curve), 6 (grey, shaded curve) and 20 (blue line) μm . The modulation frequency is taken to be $\Omega = 4.72$ rads/fs.

As a first observation, the photon numbers are now many orders of magnitude higher with respect to the single $n(t)$ -ramp case. The photon counts/second obtained by integrating over the shown spectra are $\sim 10^{12}$ and 10^{11} for emission in the x and z directions, respectively. If we then consider that we may generate intense, periodic pulse trains as described above with durations of 1-10 nanoseconds, we estimate a total of $10^2 - 10^3$ photons/pulse. Such photon numbers are well within reach of current photon counting technologies.

We note moreover that the emission in the x direction shows clear evidence of a resonance effect induced by the hard-edge boundary of the square film that is acting as a weak cavity of sorts. The larger the film, the narrower the bandwidth of the photon emission peak. Notably, this peak is always centred at the same frequency, $\omega = 2.36$ rads/fs that is determined not by the film dimensions but rather, by the dynamical Casimir resonance condition $\omega_1 = \omega_2 = \Omega/2$, where the condition $\omega_1 = \omega_2$ arises from the choice

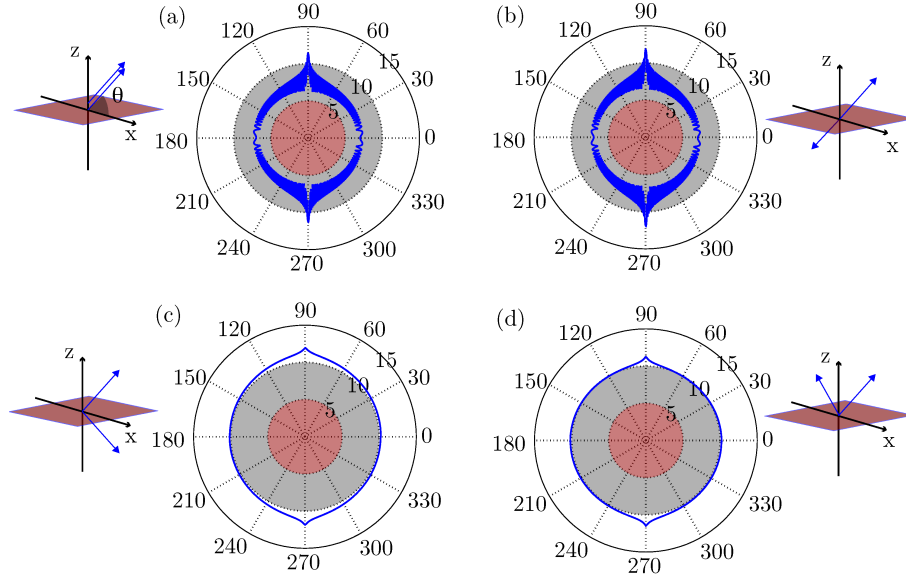


Figure 4. Polar plots showing the angular distribution of the photon emission (indicated in total photon numbers/second, log scale). (a) Photon pairs emitted with the same x and z components, (b) Photon pairs with opposite x and z (back to back), (c) Photon pairs with same x and opposite z , (d) Photon pairs with opposite x and same z .

of considering only back to back photon emission. Emission in the z direction is also peaked at the same $\Omega/2$ frequency although the subwavelength dimensions of the film do not enforce any cavity-like effects and hence leads to remarkably broadband emission. In Fig. 4 we show the angular emission pattern for four different cases, as indicated by the schematic drawing next to each of the four polar plots. The polar angle is measured such that 0 deg corresponds to emission along the x direction and 90 deg along the z direction. An interesting observation is the clear angular pattern that occurs when the photon emission is either such that both photons are emitted exactly in the same direction or exactly back to back: in both cases oscillations appear in the angular emission due to interference between the emission probabilities of the two photons that fall in and out of phase as the angle is changed. Maximum emission for all cases is observed in the z direction.

5. Gravitational wave amplifier for photons.

In the previous section we analysed the photon production from a medium whose refractive index varies as $n(t) = n_0 + \delta n \cos(\Omega t)$ in the lab frame and the relevant spacetime explicitly takes the form

$$ds^2 = c^2 dt^2 - dx^2 [n_0 + \delta n \cos(\omega t)]^2. \quad (7)$$

Assuming that $\delta n \ll 1$ we find that the null geodesics describing photon trajectories are described by the equation

$$\frac{dx}{dt} \simeq \frac{c}{n_0} \left[1 - \frac{\delta n}{n_0} \cos(\omega t) \right]. \quad (8)$$

At this point it would seem that systems with similar metrics should also have similar quantum emission properties, and now we can use this to look for other interesting structures.

The periodically modulated δn gives rise to a metric with an oscillating velocity for the ‘flow of space’, or in other words, a gravitational pull that oscillates periodically.

Gravitational waves were first predicted in 1916 by Albert Einstein as ripples in spacetime that propagate in the underlying spacetime, and is thought to be a consequence of the Lorentz invariance of general relativity; the speed of which information about fields travel cannot exceed the speed of light. These ripples are created by numerous binary systems, including those of black holes or neutron stars and carry away energy from the orbit. However, as of this day, there has not been any direct observation of a gravitational wave, despite many years of experimental trial and increasingly sensitive detectors.

The interaction of gravitational waves with other particles, e.g. bosons, has attracted significant attention in the past. G. Gibbons first predicted that a gravitational wave will not excite massive particles from the vacuum state [19], although by imposing adequate boundary conditions e.g. a cavity, particle production may be expected [20, 21]. The specific case of interaction with photons has also been considered. Similarly to massive particles, photons were explicitly predicted to not be created by a gravitational wave [22] but the presence of certain boundary conditions can lead to the creation of electromagnetic waves, e.g. in the presence of a background magnetic field or plasma [23, 24].

Here we consider the case of gravitational wave passing through a limited “slice” of space, thus isolating a region similarly to the dielectric film considered above. So if we choose to look at the case when we have gravitational waves propagating in the z -direction on a flat spacetime background, the resulting metric takes the following form:

$$\begin{aligned} ds^2 = & c^2 dt^2 - dx^2 \left[1 + h_+ \cos\left(\omega\left(t - \frac{z}{c}\right)\right) \right] \\ & - dy^2 \left[1 - h_+ \cos\left(\omega\left(t - \frac{z}{c}\right)\right) \right] - 2h_x \cos\left(\omega\left(t - \frac{z}{c}\right)\right) dx dy \end{aligned} \quad (9)$$

where h_+ and h_x denote the gravitational wave amplitudes and are related to the polarisation of the wave. We now take special interest in the case where only t and x changes. The metric then takes the form

$$ds^2 = c^2 dt^2 - dx^2 [1 + h_+ \cos(\omega t)] \quad (10)$$

and under the assumption that $h_+ \ll 1$, we arrive at a similar formula to Eq. (8) for photon trajectories,

$$\frac{dx}{dt} = \frac{c}{\sqrt{1 + h_+ \cos(\omega t)}} \simeq c \left(1 - \frac{h_+}{2} \cos(\omega t) \right) \quad (11)$$

The similarity to Eq. (8) implies that we have a system where light experiences an x coordinate that periodically changes in time in a nearly identical manner. Rather than attempt to derive a full quantum treatment of the problem, here we limit the analysis to demonstrating that gravitational waves may act as amplifiers for electromagnetic waves. This demonstration relies solely on the kinematic analogy of the spacetime contraction created by a gravitational wave and a refractive index change in an optical medium. Nevertheless, this simplified approach allows to show the amplification properties of gravitational waves which must therefore also apply to quantum fluctuations of the electromagnetic field.

Our treatment follows closely the same approach proposed by Mendonça et al., in the context of what has been called “time refraction”, i.e. the interaction of light with a time varying boundary at a fixed position in space [25]. The displacement tensor $D^{\alpha\beta} = \frac{\sqrt{-g}}{\mu_0} F^{\alpha\beta}$, where $F^{\alpha\beta}$ is the electromagnetic field tensor and $\sqrt{-g}$ is the determinant of the metric $g_{\alpha\beta}$. The implication of this result is that for some simple metrics $g_{\alpha\beta}$, $\sqrt{-g}$ can be used as an equivalent dielectric constant. In other words, here we are taking an opposite approach to the rest of this manuscript in the sense that rather than try to use an effective $n(t)$ to mimic a cosmological system, we are now asking taking a cosmological system, i.e. a propagating gravitational wave. We are then attempting an albeit simplified description that is implicitly based on thinking in terms of an effective refractive index modulation. In our case the metric will take the form $ds^2 = c^2 dt^2 - \eta(t)^2 dz^2$.

Now imagine that at $t = 0$ we suddenly change η as a step function. Then Maxwell’s equations tell us that both the displacement field and its time derivative will be continuous at the boundary. Hence (in three-vector notation) the field before the step

$$D_1(t) = \eta_1 E_i e^{-i\omega_1 t} \quad (12)$$

$$\frac{\partial D_1(t)}{\partial t} = -i\omega_1 \eta_1 E_i e^{-i\omega_1 t} \quad (13)$$

has to be continuous with the field after the step.

$$D_2(t) = \eta_2 E_r e^{i\omega_j t} + \eta_2 E_t e^{-i\omega_j t} \quad (14)$$

$$\frac{\partial D_2(t)}{\partial t} = i\omega_2 \eta_2 E_r e^{i\omega_2 t} - i\omega_2 \eta_2 E_t e^{-i\omega_2 t}. \quad (15)$$

Thus

$$\eta_1 E_i = \eta_2 E_r + \eta_2 E_t \quad (16)$$

$$\omega_1 \eta_1 E_i = -\omega_2 \eta_2 E_r + \omega_2 \eta_2 E_t. \quad (17)$$

Now we make use of the relation $\omega_1 \eta_1 = \omega_2 \eta_2$ and by defining $\zeta = \eta_1/\eta_2$ we get

$$\zeta E_i = E_r + E_t \quad (18)$$

$$E_i = -E_r + E_t \quad (19)$$

and it follows that

$$E_t = \frac{\zeta + 1}{2} E_i \quad (20)$$

$$E_r = \frac{\zeta - 1}{2} E_i \quad (21)$$

which defines the transmission and reflection coefficients for the temporal step. With this in mind we can look at what happens if there is a second, successive 'step' in $\eta(t)$ such that $\zeta' = \eta_2/\eta_3$ and that occurs after some time τ . The resulting field then has the form, separated into components with same time

$$E_f = E_i[TT'e^{-i\omega_2\tau} + RR'e^{i\omega_2\tau}]e^{-i\omega_3t} \quad (22)$$

$$E'_f = E_i[RT'e^{i\omega_2\tau} + TR'e^{-i\omega_2\tau}]e^{i\omega_3t} \quad (23)$$

where R, R' and T, T' refer to the reflection and transmission coefficients respectively for each (temporal) boundary. Expanded, these relations take the form

$$E_f = \frac{E_i}{4}[(\zeta + 1)(\zeta' + 1)e^{-i\omega_2\tau} + (\zeta - 1)(\zeta' - 1)e^{i\omega_2\tau}]e^{-i\omega_3t} \quad (24)$$

$$E'_f = \frac{E_i}{4}[(\zeta - 1)(\zeta' + 1)e^{i\omega_2\tau} + (\zeta + 1)(\zeta' - 1)e^{-i\omega_2\tau}]e^{i\omega_3t} \quad (25)$$

which if we assume $\zeta' = 1/\zeta$, reduces to

$$E_f = \left[\cos(\omega_2\tau) - \frac{i}{2\zeta}(1 + \zeta^2) \sin(\omega_2\tau) \right] E_i e^{-i\omega_1t} \quad (26)$$

$$E'_f = \left[\frac{i}{2\zeta}(1 - \zeta^2) \sin(\omega_2\tau) \right] E_i e^{i\omega_1t}. \quad (27)$$

Note the exact resemblance to time-refraction. Also, if we now set τ such that $\omega_2\tau = (2m + 1)\pi/2$ with $m = 0, 1, 2, \dots$ then we can see that the new wave has higher amplitude than the incident wave; the electromagnetic wave is amplified. Furthermore, if we presume that $\zeta = 1 + \epsilon$ where $\epsilon \ll 1$, then the equations reduce to

$$E_f \simeq -iE_i e^{-i\omega_1t} \quad (28)$$

$$E'_f \simeq -i\epsilon E_i e^{i\omega_1t}. \quad (29)$$

The total energy has increased by factor ϵ . Suppose now that we have $n - 1$ additional square steps, then one can show that the fields take the form

$$E_f \simeq (-i)^n \sum_{k=0}^n \binom{n}{2k} \epsilon^{2k} E_i e^{-i\omega_1t} \quad (30)$$

$$E'_f \simeq (-i)^n \sum_{k=0}^n \binom{n}{2k+1} \epsilon^{2k+1} E_i e^{i\omega_1t}. \quad (31)$$

Thus the total wave energy grows as $(1 + \epsilon)^n$. For an electromagnetic field interacting with a square gravitational wave oscillating at $\Omega = \pi/\tau$, then n is simply the number of half-cycles elapsed during time t and thus the total electromagnetic wave energy grows exponentially in time: $W(t) \simeq (1 + \epsilon)^{n(t)} W_0$, where W_0 is the initial energy.

A similar model may be developed accounting for the quantum nature of photons and therefore describing the interaction with the vacuum modes. The full calculation conceptually follows the same steps outlined above and once again is very similar to

quantum treatment of time refraction developed by Mendonça et al. [26]. The output photon modes are related to the input modes by the standard Bogoliubov transformation

$$\hat{a}_{\text{out}}(k) = \alpha \hat{a}_{\text{in}}(k) - \beta \hat{a}_{\text{in}}^\dagger(-k) \quad (32)$$

$$\hat{a}_{\text{out}}^\dagger(-k) = \alpha \hat{a}_{\text{in}}^\dagger(-k) - \beta \hat{a}_{\text{in}}(k) \quad (33)$$

where \hat{a} and \hat{a}^\dagger are the photon creation and annihilation operators with Bogoliubov coefficients, α and β that for a square shaped contraction followed by a square shaped expansion (i.e. one period of the oscillation) that are given by

$$\alpha = \frac{i}{2\zeta}(\zeta^2 + 1)e^{-i\omega_1 t} \quad (34)$$

$$\beta = \frac{i}{2\zeta}(\zeta^2 - 1)e^{i\omega_1 t}. \quad (35)$$

The simple fact that these coefficients are different from zero is sufficient to conclude that indeed photons will be amplified and/or excited out of the vacuum state by a gravitational wave. Clearly a more complete model is required in order to evaluate precisely how efficient the process will be. However, it is possible to gain some insight into the process without resorting to more complicated calculations.

A first observation regards the precise geometry required to excite photons out of the vacuum state: although not explicitly stated, the model adopted here relies on the assumption that the medium, (i) varies uniformly over its entire extent and, (ii) that only one compression or expansion event occurs at a time over the whole sample, i.e. the sample must be shorter in the z direction than half the period of the gravitational wave. This is in keeping with the model and assumptions of the thin film with a periodic refractive index. Also this must be chosen such that the same conditions are met. If the film or region of spacetime affected by the contraction/expansion is thicker than the oscillation period then the average total effect will go to zero.

One must therefore limit a region of space along the propagation direction of the gravitational wave, e.g. by adopting a cavity geometry. This would lead to a configuration identical to the original proposal for measuring the dynamical Casimir effect, i.e. the emission of photons from a periodically modulated cavity. In this case, the cavity length would be modulated by the gravitational wave. However, radiation from such a simple geometry has little hope of being directly observed and indeed all modern attempts to study dynamical Casimir radiation rely on other systems in which the cavity may be modulated at very high frequencies and amplitudes [12, 27, 28]. Gravitational waves are extremely weak and the change in the cavity length induced in Earth based cavities would be of the order of 1 part in 10^{-20} or less with frequencies in the kHz range. It is therefore very unlikely that such an experiment would therefore be successful.

6. Concluding remarks.

Optical nonlinearities in very thin transparent optical media excited by a laser pulse or a train of laser pulses will give rise to a refractive index that varies in time, uniformly across the sample. Such a geometry can model a cosmological expansion or contraction and can thus be used to study details of how photons may be excited in similar spacetime metrics. A single contraction/expansion appears to deliver a very weak flux of photon pairs. However, the flux may be considerably enhanced by performing a periodic modulation, in which case a resonance is observed in the emitted photon frequencies corresponding to half the modulation frequency. The same spacetime metric bears a similarity to a gravitational wave which will also amplify electromagnetic waves. However, this will occur only if boundary conditions are imposed, e.g. confinement within a cavity, that limit the interaction of the gravitational wave with the vacuum to a region that is smaller than the gravitational perturbation wavelength. It seems that direct detection of actual gravitational waves based upon photon excitation in a cavity is looking unlikely due to the extreme weakness of the gravitational waves themselves, although different settings based e.g. on Bose-Einstein condensates may still hold promise [21]. The analogue dielectric medium model proposed here however, does allow a greater degree of control over the experimental conditions and thus provides an intriguing possibility to study the quantum features of these and other cosmological effects using laboratory based experiments.

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